

PURBANCHAL UNIVERSITY

I SEMESTER FINAL EXAMINATION-2007

LEVEL: Bachelor of Technology in Biotechnology

SUBJECT: Mathematics-I

TIME: 03:00 hrs.

Full Marks: 80

Pass Marks: 32

Candidates are required to give their answers in their own words as far as practicable.

Answers in the margin indicate full marks.

GROUP-A

Answer FOUR questions.

[4×10=40]

[1] Find the area bounded by $x^2=4y$ and the lines $x=4y-2$.

[2] Find the asymptotes of the curve

$$4x^3 - 3xy^2 - y^3 + 2x^2 - xy - y^2 - 1 = 0.$$

[3] Find the radius of curvature at the point (x, y) for the curve $y=c \cosh \frac{x}{c}$.

[4] Evaluate the double integral $\iint xy(x+y) dx dy$, over the region bounded by the curve; $y=x$ and $y=x^2$.

[5] If $u = \sqrt{x^2 + y^2 + z^2}$, prove that $\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2 + \left(\frac{\partial u}{\partial z}\right)^2 = 1$.

GROUP-B

Answer SIX questions.

[6×5=30]

[6] If $y = (x^2 - 1)^n$, prove that

$$(x^2 - 1)y_{n+2} + 2xy_{n+1} - n(n+1)y_n = 0.$$

Contd. ...

(2) $a = \pi/2$

Q. [7] Expand $\sin x$ in power of $(x - \pi/2)$.

Q. [8] Find the reduction formula for $\int \sin^n x dx$.

Q. [9] If $u = f(y - z, z - x, x - y)$ show that $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0$.

Q. [10] Evaluate $\int_0^1 \int_0^{1-x} \int_0^{1-x-y} dz dy dx$. $(1/3)$

Q. [11] Test the convergence of the series

$$\frac{1}{2\sqrt{1}} + \frac{x^2}{3\sqrt{2}} + \frac{x^4}{4\sqrt{3}} + \dots \infty$$

Q. [12] Find the interval of convergence of the series: $\sum_{n=1}^{\infty} \frac{n(x+2)^n}{3^n + 1}$.

Q. [13] If a is a constant vector and $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$, prove that $\text{curl}(\vec{a} \times \vec{r}) = 2\vec{a}$.

Q. [14] Verify Euler's theorem for the function $u = \frac{x(x^3 - y^3)}{x^3 + y^3}$.

GROUP-C

Answer ALL questions.

[5×2=10]

Q. [15] State Euler's theorem.

Q. [16] Define absolute and conditional convergence.

Q. [17] Evaluate $\int_0^{\pi/2} \sin^3 \theta \cos^4 \theta d\theta$. $\frac{2}{3} \frac{\pi^{1/2}}{\pi^{1/2}} = \frac{2 \times \frac{1}{2} \times \frac{1}{2}}{2 \times \frac{1}{2} \times \frac{1}{2}}$

Q. [18] Given $\phi(x, y, z) = xy + yz + zx$, find grad ϕ at the point $(1, 2, 3)$.

Q. [19] Find $\nabla \times (x^2\vec{i} + y^2\vec{j} + z^2\vec{k})$.

PURBANCHAL UNIVERSITY

2008

Bachelor of Technology in Biotechnology / First Semester / Final

Time: 03:00 hrs.

Full Marks: 80 / Pass Marks: 32

Mathematics-I

Candidates are required to give their answers in their own words as far as practicable.

Figures in the margin indicate full marks.

Group A: Long-answer Questions

Answer FOUR questions.

4 × 10 = 40

If $y = (\sin^{-1} x)^2$ prove that

10

$$(1 - x^2) y_{n+2} - (2n + 1) x y_{n+1} - x^2 y_n = 0$$

If $u = \tan^{-1} \left(\frac{x^3 + y^3}{x - y} \right)$ prove that $Xu_x + Yu_y = \sin 2u$

Find the asymptotes of the following curve:

$$x^3 + 2x^2y - xy^2 + 2y^3 + 4y^2 + 2xy + y - 1 = 0$$

Change the order of integration and, hence evaluate:

$$\int_0^{4a} \int_{x^2/4a}^{2\sqrt{ax}} dy dx$$

Find the area bounded by the parabola $y^2 = 9x$ and the line

$$y = 3x.$$

Group B: Short-answer Questions

Answer SIX questions.

6 × 5 = 30

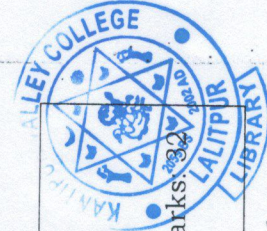
Find the radius of curvature for the curve defined by

$$x = 3/t^2, y = 1/t \text{ at the point } (0, 0).$$

Using Leibnitz theorem, find the second derivative of

$$y = e^x \cos x.$$

Contd. ...



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8. Show that $\int_0^x \sin^n x dx = \left(\frac{n-1}{n} \right) \int_0^x \sin^{n-2} x dx$. Use this formula

to compute $\int_0^{\pi/2} \sin^4 x dx$.

9. Expand $\sin \left(\frac{\pi}{4} + \theta \right)$ in a series of powers of θ .

10. If $f(x) = x^3 + x$, show that there is a point c in $[0, 1]$ such that $f'(c) = 0$.

11. Change the order of integration and hence, evaluate:

$$\int_0^{2a} \int_{-4a}^{2a-y} (x^2 + y^2) dx dy$$

12. Examine the convergence of the series: $\sum_{n=1}^{\infty} \sqrt{\frac{x}{x+1}} x^n$.

13. Find the interval of convergence of the series: $\sum_{n=1}^{\infty} \frac{(x-2)^n}{n^2}$.

14. Calculate: $\nabla^2 \left(\sqrt{x^2 + y^2 + z^2} \right)$

Group C: Very Short-answer Questions

Answer FIVE questions.

5 × 2 = 10

15. Verify Euler's theorem for $u = e^{x/y}$.

16. Find the value of c so that $\vec{V}(x + 2y)\vec{i} + (y + 3z)\vec{j} + (y + cz)\vec{k}$ is solenoidal.

17. State Cauchy's root test.

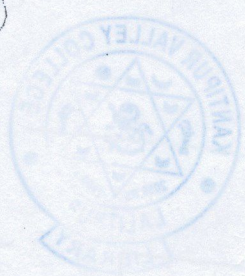
18. Define curvature and the radius of curvature.

19. Evaluate: $\int x^2 (1 - x^2)^4 dx$

20. What point on the parabola $y = x^2 + x + 1$ satisfies the mean value theorem in $[-1, 3]$.

21. Define gradient, divergence and curl.

10



PURBANCHAL UNIVERSITY
2009

Bachelor of Technology in Biotechnology / First Semester / Final
Full Marks: 80 / Pass Marks: 32

Time: 03:00 hrs.

Mathematics-I

Candidates are required to give their answers in their own words as far as practicable.
Figures in the margin indicate full marks.

Group A: Long-answer Questions

4x10=40

Answer **FOUR** questions.

1. Find the area bounded by $y = \sin x$, the axis of x and the ordinate $x = 0$ and $x = 2\pi$.
2. Find the asymptotes of the curve:
 $4x^3 - 3xy^2 - y^3 + 2x^2 - xy - y^2 - 1 = 0$.
3. Find the radius of curvature at the point (x, y) of the curve $y^2 = x^3$.
4. Check the order of the integration and hence evaluate:
$$\int_0^1 \int_0^1 \frac{e^{xy}}{y} dy dx.$$
5. Verify Euler's theorem for the function $f(x, y) = \sqrt{x^2 + xy + y^2}$.

Group B: Short-answer Questions

6x5=30

Answer **SIX** questions.

1. Using Leibnitz theorem, find the n th derivative of $y = x^3 e^{2x}$.
2. Expand $\sin x$ in the power of $\left(x - \frac{\pi}{2}\right)$.
3. Find the reduction formula for $\int \sin^n x dx$.
4. If $u = \sqrt{x^2 - y^2 + z^2}$ prove that $\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2 + \left(\frac{\partial u}{\partial z}\right)^2 = 1$.
Contd. ...

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PURBANCHAL UNIVERSITY
2012

Bachelor of Technology in Biotechnology / First Semester / Final
Time: 03:00 hrs. Full Marks: 80 / Pass Marks: 32

Mathematics-I

Candidates are required to give their answers in their own words as far as practicable.

Figures in the margin indicate full marks.

Group A:

Answer FOUR questions.

4×10=40

1. Find the area bounded by $y = x^2 + x + 2$ and the lines $x = -2$, $x = 1$ and $x = 0$.
2. Find the asymptotes of the curve $4x^3 - 3xy^2 - y^3 + 2x^2 - xy - y^2 - 1 = 0$
3. Find the radius of curvature of (r, θ) of $r = a \cos \theta$.
4. Change the order of integration and hence evaluate the double integral $\int_1^2 \int_3^4 (x^2 + y^2) dx dy$.

5. If $u = \sqrt{x^2 + y^2 + z^2}$, prove that $\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2 + \left(\frac{\partial u}{\partial z}\right)^2 = 1$.

Group B:

Answer SIX questions.

6×5=30

6. If $y = (\sin^{-1} x)^2$, prove that $(1 - x^2)y_{n+2} - (2n+1)xy_{n+1} - n^2 y_n = 0$.
7. Expand by Maclaurin's series of $f(x) = e^x \cos x$.
8. Find the reduction formula for $\int \sin^n x dx$.
9. For the function $f\left(\frac{z}{x}, \frac{x}{y}\right) = 0$, show that $y \frac{\partial x}{\partial y} + z \frac{\partial x}{\partial z} = x$.

Contd. ...

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10. Evaluate $\int_0^a \int_0^x \int_0^y xyz \, dx \, dy \, dz$.

11. State Leibnitz Test. Test the convergence of the series

$$\sum_1^{\infty} \left(\frac{nx}{n+1} \right)^n.$$

12. Find the interval of convergence of the series $\sum_1^{\infty} \frac{(x+2)^n}{n^2}$.

13. Prove that $\vec{v} = yz\vec{i} + zx\vec{j} + xy\vec{k}$, is a solenoidal.

14. Verify Euler's theorem for $u(x,y) = \sin^{-1} \left(\frac{x+y}{x-y} \right)$.

Group C:

Answer ALL questions.

5×2=10

15. State root test.

16. Verify Lagrange mean value theorem for $f(x) = e^x, [0,1]$

17. Evaluate $\int_0^{\pi/2} \sin^3 \theta \cos^4 \theta \, d\theta$.

18. If $\phi(x, y, z) = xy + yz + zx$, find $|\nabla \phi|$.

19. If $y = x^n$, find y_n

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PURBANCHAL UNIVERSITY**2013**

Bachelor of Technology in Biotechnology / First Semester / Final

Time: 03:00 hrs.

Full Marks: 80 / Pass Marks: 32

BT101SH; Mathematics-I

Candidates are required to give their answers in their own words as far as practicable.

Figures in the margin indicate full marks.

Group A**Answer FOUR questions.****4×10=40**

1. Find the asymptotes of the curve $(x^2 - y^2)^2 - 2(x^2 + y^2) + x - 1 = 0$
2. If $\log y = \tan^{-1} x$ then show that
 - (a) $(1 + x^2)y_2 + (2x - 1)y_1 = 0$
 - (b) $(1 + x^2)y_{n+2} + (2xn + 2x - 1)y_{n+1} + n(n+1)y_n = 0$
3. Find the area bounded by the parabola $y = x^2 + 2$ and the lines $x = 0$, $x = 1$ and $x + y = 0$.
4. Change the order of integration and hence evaluate the double integral $\int_0^4 \int_{\sqrt{y}}^2 \sqrt{x^3 + 1} dx dy$
5. If $u = \tan^{-1} \frac{x^2 + y^2}{x - y}$ show that $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = \frac{1}{2} \sin 2u$.

Group B**Answer SIX questions.****6×5=30**

6. Expand $\log(1 + x)$ by Maclaurin's theorem as far as the term containing x^5 .
7. State ratio test and determine if $\sum_{n=1}^{\infty} \frac{2^n}{n!}$ converges or diverges.
8. Find the interval of the convergence of the series $\sum_{n=0}^{\infty} \frac{(-1)^n (x+1)^n}{2^n}$.

Contd. ...

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9. Find the radius of curvature at the point (r, θ) of the curve:
 $r = a(1 - \cos\theta)$.

10. Find the reduction formula for $\int \cos^n x dx$.

11. Use Gamma function to evaluate $\int_0^\infty e^{-x^2} dx$.

12. Show that the vector $\vec{v} = yz\vec{i} + zx\vec{j} + xy\vec{k}$ is solenoid. Also find $\text{Curl } \vec{v}$.

13. Given $\phi(x, y, z) = xy + yz + zx$, find $\text{grad } \phi$ at the point $(1, 2, 3)$.

Group C

Answer ALL questions.

5×2=10

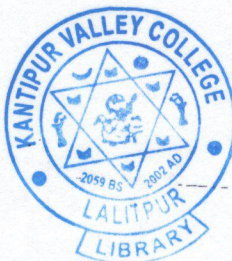
14. If $y = e^x$ find y_n .

15. Define gradient and curl.

16. State Euler's theorem for the function of two independent variables.

17. State comparison test for the convergence of the infinite series.

18. Evaluate the integral $\int_1^2 \int_1^2 xy dx dy$.



PURBANCHAL UNIVERSITY

2015

Bachelor of Technology in Biotechnology/First Semester/Final
Time: 03:00 hrs.

Full Marks: 80 /Pass Marks: 32

BT101SH: Mathematics-I (New/Old Course)

Candidates are required to give their answers in their own words as far as practicable.

Figures in the margin indicate full marks.

Group A

Answer FOUR questions.

1. Show that the radius of curvature for the curve $r^m = a^m \cos m\theta$ is **4×10=40**

$$\frac{a^m}{(m+1)r^{m-1}}$$
2. Find the interval of convergence and radius of convergence of the series $\sum_{n=1}^{\infty} \frac{2^n (x-3)^n}{n+3}$
3. If $I_n = \int_0^{\pi/4} \tan^n x dx$ then show that $I_n + I_{n-2} = \frac{1}{n-1}$ and hence deduce the value of I_5 .
4. Find the extreme values for the function $x^2 + y^2 + z^2$ subject to the condition $x + y + z = 3a$.
- 5(a) If $u = x + y + z$, $v = x^2 + y^2 + z^2$ and $w = xy + yz + zx$ then show that $[\text{grad } u, \text{grad } v, \text{grad } w] = 0$
- (b) Determine the constant a so that the vector $\vec{v} = (x + 3y)\vec{i} + (y - 2z)\vec{j} + (x + az)\vec{k}$ is solenoidal.

Group B

Answer SIX questions.

6. Verify Lagrange Mean Value theorem for the function **6×5=30**

$$f(x) = x(x-1)(x-2), x \in \left[0, \frac{1}{2}\right]$$

Contd. ...



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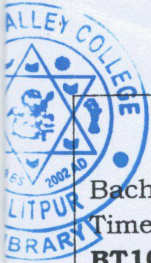
7. ✓ If $y = (\sin^{-1} x)^2$ then show that
 $(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - n^2y_n = 0.$
8. ✓ Find the asymptotes of the curve $y^3 - xy^2 - x^2y + x^3 + x^2 - y^2 - 1 = 0.$
9. ✓ Test the convergence of the series $\frac{1}{1.2.3} + \frac{3}{2.3.4} + \frac{5}{3.4.5} + \dots$
10. Using Gamma function, show that $\int_0^{\pi} 2 \sin^6 \frac{x}{2} \cdot \cos^8 \frac{x}{2} = \frac{5\pi}{2^{11}}.$
11. Find the volume of the solid generated by revolution of the cardioid $r = a(1 - \cos\theta)$ about the initial line.
12. ✓ If $u = \sin^{-1} \frac{x^2+y^2}{x+y}$, so that $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = \tan u$
13. ✓ Find by double integration, the area bounded by the parabola $y = x^2$ and the line $y = x.$

Group C

Answer ALL questions.

5×2=10

14. ✓ State Roll's theorem.
15. ✓ State p-series.
16. ✓ Evaluate $\int_0^1 \int_0^2 \int_1^2 dx dy dz.$
17. ✓ Use Leibnitz formula to evaluate $\frac{d}{dx} \int_x^{\pi^2} \cos u^2 du.$
18. ✓ Find the unit tangent vector to the curve
 $\vec{r}(t) = t^2 \vec{i} + 2 \sin t \vec{j} + 2 \cos t \vec{k}$



PURBANCHAL UNIVERSITY

2016

Bachelor of Technology in Biotechnology / First Semester / Final

Time: 03:00 hrs.

Full Marks: 80 / Pass Marks: 32

BT101SH: Mathematics-I (New/Old Course)

Candidates are required to give their answers in their own words as far as practicable.

Figures in the margin indicate full marks.

Group A

Answer FOUR questions.

4×10=40

1. State the Leibnitz's theorem. Using it, if $y = e^a \sin^{-1} x$, then prove that $(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - (x^2+a^2)y_n = 0$
2. Find the asymptotes of the curve series.
 $(x^2-y^2)^2 - 2(x^2+y^2)+x-1 = 0$
3. Show that the radius of curvature at point (r, θ) for the curve $r = ae^{\theta \cot \alpha}$ is $P = r \operatorname{cosec} \alpha$
4. Find the area above the x-axis included between the parabola $y^2 = ax$ and the circle $x^2+y^2 = 2ax$
5. Determine the radius of convergence and interval of convergence for the following power series $\sum_{n=1}^{\infty} \frac{2^n (4x-8)^n}{n}$.

Group B

Answer SIX questions.

6×5=30

6. Show that $u(x, y) = x^3 + y^3 - 3xy$ has a saddle point at $(0, 0)$.
7. If $I_n = \int_0^{\pi} \tan^n x dx$ then show that $I_n + I_{n-2} = \frac{1}{n-1}$ and hence deduce the value of I_5 .
8. Verify Rolle's theorem for the function $f(x) = x^3 - 4x$ in $[-2, 2]$

Contd.

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9. Evaluate $\int_0^1 \int_0^{1-y} \int_0^{1-x^2-y^2} dx dy dz$

10. By Using Euler's theorem, prove that $x \frac{\partial v}{\partial x} + y \frac{\partial v}{\partial y} = 2$, where

$$V = \log \frac{x^3 + y^3}{x + y}.$$

11. Test convergent of series $\sqrt[3]{x^3 + 1} - x$.

12. If $\vec{r} = (x, y, z)$ and, $\vec{a}, \vec{b}$ are vectors, prove that

$$\text{Curl } [\vec{r} \times (\vec{a} \times \vec{b})] = 2\vec{b} \times \vec{a}$$

13. Use Gamma function to evaluate

$$\int_0^{\pi/2} \sin^3 x \cos^5 x dx$$

Group C

Answer ALL questions.

5×2=10

14. Define gradient and curl.

15. State the Leibnitz's rule for differentiation under integral sign.

16. State Cauchy's n^{th} root test for the convergence of the infinite series.

17. Define curvature and the radius of curvature.

18. If $y = e^x$ find y_n .

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PURBANCHAL UNIVERSITY

2018

Bachelor of Technology in Biotechnology / First Semester / Final

Time: 03:00 hrs.

Full Marks: 80 / Pass Marks: 32

BT101SH: Mathematics-I (New/ Old Course)

Candidates are required to give their answers in their own words as far as practicable.

Figures in the margin indicate full marks.

Group A

Answer FOUR questions.

4×10=40

1. ✓ If $x = \sin\left[\frac{\log y}{a}\right]$ then prove that,

$\phi_a(m)$

(i) $(1-x^2)y_2 = xy_1 = a^2 y$.

(ii) $(1-x^2)y_{n+2} - (2n+1)y_{n+1} - (x^2 + a^2)y_n = 0$

2. The rectangular co-ordinates (x,y,z) of a point are given in terms of its spherical co-ordinates (r,θ,φ) by the $x = r \sin \theta \cos \phi, y = r \sin \theta \sin \phi, z = r \cos \theta$. Prove that the Jacobian of the transformation is $r^2 \sin \theta$.

3. Find the reduction formula for $\int \cos^m x \sin nx \, dx$.

① Divergence of $\vec{B}$ ② curl of $\vec{B}$ ③ $\vec{B}$ in r, θ, ϕ ④ $\vec{B}$ in r, θ, ϕ

4. ✓ If $\vec{f} = (x+y+z)\vec{i} + \vec{j} + (-x-y)\vec{k}$.

5(a) ✓ Prove that the series $x + \frac{x^2}{2} + \frac{x^3}{3} + \frac{x^4}{4} + \dots (x \geq 0)$ is convergent if

$0 \leq x < 1$ and divergent if $x \geq 1$.

Group B

Answer SIX questions.

6×5=30

6. ✓ Find the asymptotes of the given curve $y^2(x^2 - a^2) = x^2(x^2 - 4a^2)$.

Contd. ...

(2)

- ✓ 7. Suppose that $z=f(x,y)=x \sin y$ where $x=t^2$ and $y=2t+1$. Find dz/dt .
- ✓ 8. Find the extreme value for the function x^2+y^2 under the condition $x+4y=2$.
- ✓ 9. Prove that $\Gamma\left(-\frac{3}{2}\right) = \frac{4\sqrt{\pi}}{3}$.
10. In the catenary, $y = a \cosh(x/a)$, find the radius of curvature.
11. Find the directional derivative of $\phi=xyz$ at the point $(1,2,3)$ in the direction of $\vec{i}$.
12. Show that the volume of a right circular cone of height h and base of radius a is $\frac{1}{3} \pi a^2 h$.
- ✓ 13. If $f=x^3+y^3+z^3-3xyz$, find $\text{div}(\text{grad } f)$ and $\text{curl}(\text{grad } f)$.

Group C

Answer ALL questions.

5×2=10

14. State Roll's theorem.

15. Define power series.

16. Define improper integral.

17. Evaluate

$$\iiint_R (x^2 + y^2 + z^2) dz dy dx, \text{ where } R : 0 \leq x \leq 1, 0 \leq z \leq 1.$$

18. Find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ when $z = x^3 + y^3 + 3axy$.